Q. In a ND 7% of the items are under 35 and 89% are under 63. Find μ and σ of the Distribution.

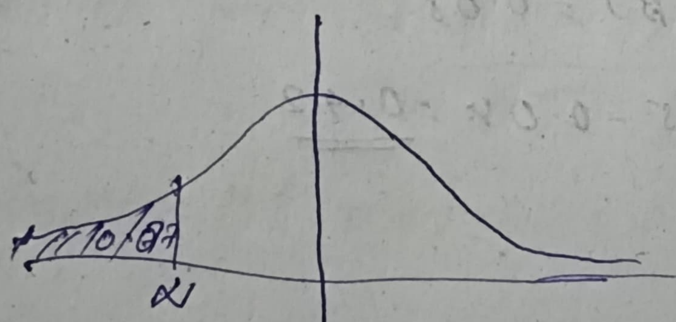
Soln: $P(x < 35) = \frac{7}{100} \rightarrow ①$

$P(x < 63) = \frac{89}{100} \rightarrow ②$

Take $z = \frac{x - \mu}{\sigma} \sim N(0,1)$

$x = 35 \Rightarrow z = \frac{35 - \mu}{\sigma} = \alpha$

$① \Rightarrow P(z < \alpha) = 0.07$



P(~~0<z<α~~) $= 0.5 - P(0 < z < \alpha) = 0.07$

$P(0 < z < \alpha) = 0.5 - 0.07$

$= 0.43$

$\alpha = -1.48$

$$\frac{35-\mu}{\sigma} = -1.48$$

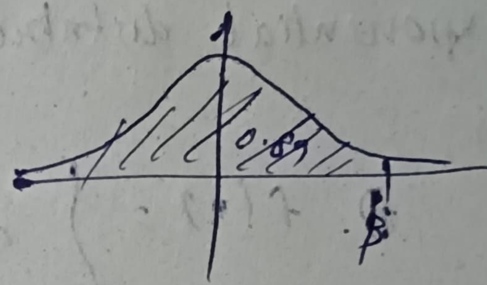
$$35-\mu = -1.48\sigma$$

$$-1.48\sigma + \mu = 35 \rightarrow (3)$$

or $1.48\sigma - \mu = -35 \rightarrow (3)$

$$x = 63 \Rightarrow z = \frac{63-\mu}{\sigma} = \beta$$

$$(2) \Rightarrow P(z < \beta) = 0.89$$



$$0.5 + P(0 < z < \beta) = 0.89$$

$$P(0 < z < \beta) = 0.89 - 0.5$$

$$= 0.39$$

$$\beta = 1.23$$

$$\frac{63-\mu}{\sigma} = 1.23$$

$$63-\mu = 1.23\sigma$$

$$1.23\sigma + \mu = 63 \rightarrow (4)$$

$$1.48\sigma - \mu = -35$$

$$2.71\sigma = 28$$

$$\sigma = \underline{10.33}$$

$$(3) \Rightarrow \mu = \underline{50.29}$$

Exponential Distribution

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

A continuous random variable x is said to follow exponential distribution with parameter λ if its pdf is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

Q. Find the mean and variance of exponential distribution

Soln: The exponential distribution $f(x) = \lambda e^{-\lambda x}, x \geq 0$

Mean $= E(X)$

$$= \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \cdot \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_{0}^{\infty} x \cdot e^{-\lambda x}\, dx$$

$$\int uv\,dx = uv' - u_1 v'' + u_2 v''' + \ldots$$

$$= \lambda \left[x \left(\frac{e^{-\lambda x}}{-\lambda} \right) - (1)\left(\frac{e^{-\lambda x}}{-\lambda x \times -\lambda} \right) \right]_0^\infty$$

$$= \lambda \left[-\frac{x e^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2} \right]_0^\infty \qquad e^{-\infty} = 0$$

$$= \lambda \left\{ [0 - 0] - \left[0 - \frac{e^0}{\lambda^2} \right] \right\}$$

$$= \lambda \times \frac{1}{\lambda^2} = \frac{1}{\lambda}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$$

$$= \int_0^\infty x^2 \cdot \lambda e^{-\lambda x}\,dx$$

$$= \lambda \cdot \left[x^2 \cdot \frac{e^{-\lambda x}}{-\lambda} - 2x \times \left(\frac{e^{-\lambda x}}{-\lambda x \times -\lambda} \right) + (2)\left(\frac{e^{-\lambda x}}{-\lambda x - \lambda \times -\lambda} \right) \right]_0^\infty$$

$= \lambda \left[(0 - 0 + 0) - \left(0 - 0 - \frac{2e^0}{\lambda^3}\right) \right]$

$= \lambda \times \frac{2}{\lambda^3} = \frac{2}{\lambda^2}$ //

$Var(x) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$

$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$ //